Relationism Rehabilitated?
I: Classical Mechanics

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Abstract

The implications for the substantivalist–relationist controversy of Barbour and Bertotti's successful implementation of a Machian approach to dynamics are investigated. It is argued that in the context of Newtonian mechanics the Machian framework provides a genuinely relational interpretation of dynamics and that it is more explanatory than the conventional, substantival interpretation. In a companion paper (Pooley 2001), the implications of the Machian approach for the interpretation of relativistic physics are explored.

1 Introduction

A brief summary of the recent history of the substantivalist–relationist debate might go as follows:

The late 1960s and the 1970s saw the rise of the modern form of substantivalism. With the demise of logical empiricism, and with the rise of scientific realism, the spacetime manifold started to be seen as a respectable entity. It appeared integral to our best physical theories and thus we were justified in postulating its existence (Earman 1970, Stein 1970). Relationist[1] critics of realistically construed spacetime, while able to offer a clear account of the relational content of substantivalist models (at least in the prerelativistic context), were exposed as

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[1]Here and throughout, we conform to the standard practice of using "relationist" to stand for someone who denies that space, or spacetime, is a basic entity, ontologically on a par with matter. An alternative use of the word also has some currency according to which "relationism" is roughly synonymous with "anti-haecceitism". For more on these two senses of the term, see Pooley (2001, Section 3.1).

unable to offer a formulation of physics which did not, at least tacitly, involve primitive spacetime structures. Consider John Earman:

> ...the absolutist claims that the laws of physics cannot be stated without the use of an apparatus that carries with it a commitment to substantivalism. In this the absolutist may be wrong, but he is right in holding that the relationist must meet the challenge of formulating a relationally pure physics. The history of relationism is notable for its lack of success in meeting this challenge. (Earman 1989, 135)

This confidence in spacetime was shaken in the 1980s when philosophers reacquainted themselves with Einstein's "Hole Argument" (Stachel 1989). It was now feared that any commitment to spacetime as a real entity involved embracing indeterminism. But surely, the argument went, the issue of determinism is to be settled by the specific details of physics, not on the basis of such a general ontological commitment (Earman & Norton 1987).

The substantivalist responses were swift and, despite some important dissenting voices, a certain unanimity has emerged: spacetime realism does not entail indeterminism. Most endorse a *sophisticated substantivalism*: they claim that the spacetime realist is entitled to and should count spacetime models related by "hole" diffeomorphisms as representing the same state of affairs (Mundy 1992, Brighouse 1994, Rynasiewicz 1994, Hoefer 1996, Bartels 1996).[2] Many claim, for example, that it is a mistake to view the bare spacetime manifold, stripped of its affine and metrical properties, as representing spacetime (Mundy 1992, DiSalle 1994, Hoefer 1996).

Two more recent developments are worthy of note. Some philosophers have started to question whether the debate truly transfers from its historical setting—of persisting matter in space—to the modern context of fields and spacetime. It is argued that it is a matter of whim whether one sees classical general relativity as vindicating Newton's absolute space or Descartes' relational ethers (Rynasiewicz 1996). Others, by contrast, now see current research programmes which attempt to unify quantum mechanics and general relativity in a coherent theory of quantum gravity as the proper testing ground for the age-old debate, arguing that relationist and substantivalist interpretations of classical general relativity are allied to different approaches to quantizing the theory (Barbour 1986, Hoefer 1998, Belot & Earman 2000, Belot & Earman 2001). Certainly many of the research workers in one of the main approaches to quantum gravity see the debate between the relationist and the substantivalist as of relevance to their enterprise (e.g. Rovelli 1997, Smolin 1991).

[2] The main dissenting voices those of Gordon Belot and John Earman (Belot 1999, Belot 2000, Belot & Earman 2001) who coin the disparaging phrase "sophisticated substantivalism". For a defence of sophisticated substantivalism in the face of Belot's and Earman's objections, see Pooley (2001, Section 3.1). Note that Butterfield (1989) and Maudlin (1990) are substantivalists who also deny the physical equivalence of diffeomorphic models but without accepting indeterminism. For different reasons they argue that only one model in each equivalence class is to be regarded as representing a genuine physical possibility.

We subscribe to the view that the debate is not outmoded (for an incisive rejoinder to Rynasiewicz's scepticism, see Hoefer 1998). The question of what constitutes the 'correct' interpretation of classical general relativity is still open and is surely one that has ramifications for contemporary physics. But in addressing it, philosophers would also be well-advised to re-examine the received wisdom concerning prerelativistic physics first. Contrary to orthodox opinion (Friedman 1983, Earman 1989, Maudlin 1993, DiSalle 1994, Rynasiewicz 1995*a*), it has been shown, by Julian Barbour and Bruno Bertotti, that in the context of prerelativistic physics relationism is a viable option (Barbour & Bertotti 1982). What little attention this work has received in the philosophical literature has, for the most part, been cursory and misleading.[3] Indeed, the relational, "Machian" theories are arguably more explanatory than their conventional rivals. The main object of this paper is to advertise this fact, and to draw out some of the philosophical consequences. Recently, aside from Barbour himself, Gordon Belot has been the principle advocate of the viability of relationism in the context of classical mechanics (Belot 1999, Belot 2000). We agree with his positive assessment. However, for reasons spelled out below, we believe that Barbour's formulation of relational mechanics is both more fundamental and more illuminating than the Hamiltonian formulation that Belot discusses.

2 Newton versus Leibniz

The true home of the substantivalist–relationist controversy is prerelativistic classical mechanics. Things are far less clear-cut as soon as one considers more contemporary branches of physics. The following brief and selective survey of the historical origins of the debate provides a context for our discussion of Barbour and Bertotti's intrinsic particle dynamics in Section 7. Our concern is to highlight the important philosophical issues, not to achieve historical accuracy.

Newton recognised that Descartes' particular relational concept of motion was inadequate for the formulation of Descartes' own law of inertia. Newton effectively postulated a preferred *equilocality relation* between the points of space at different times and a primitive measure of the *temporal 'distance'* between them in order to associate with every body an unambiguous measure of its motion. His equilocality was defined by the simple persistence of the points of space: "Absolute space, in its own nature, without relation to anything external, remains always similar and immoveable. Relative space is some moveable dimension or measure of the absolute spaces; which our senses determine by its position to bodies" (Newton 1729).

It is perhaps possible to quibble over whether Newton's points of space are

[3] As will become clear in Section 5, Gordon Belot (1999, 2000) is a notable exception. Earman also deserves much credit for bringing Barbour's early work to the attention of the philosophical community (Earman 1989, Ch. 5). However, he confines his discussion of Barbour and Bertotti's 1982 paper to a footnote and claims that "this new theory seems to amount to a reworking of the approach of Zanstra (1924)" (1989, 212, n. 5). In what follows, we hope to show that this influential assessment is far from being correct.

genuinely 'substantival' but the important point is that the Newtonian equilocality relation is determined entirely independently of the nature of matter and its relative motion. That space and duration have their 'own natures' which are not determined by 'anything external' (i.e., by *matter* in any shape or form) clearly makes Newton some brand of spacetime realist.[4]

Leibniz rejected the reality of both of these aspects of space and time. In his third reply to Clarke, he writes:

> As for my own opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be the order of coexistences, as time is an order of successions. For space denotes, in terms of possibility, an order of things which exist at the same time. (Alexander 1956, 25–26)

For Leibniz there was nothing more to space than the fact that the spatial relations of any possible instantaneous extended material configuration—the relative distances between material objects ("things") at any given instant—possess a very particular order: they conform to Euclidean geometry. Similarly, time is nothing more than the linear succession of such highly ordered instantaneous material configurations.

3 Absolute space versus an affine connection

Relationists were quick to seize upon a problematic aspect of Newton's postulation of a *single*, privileged equilocality relation. From the point of view of classical mechanics there is an infinite family of such relations (given by the world lines of the points of the relative spaces of the full family of inertial frames) which are each fully capable of doing the job of Newton's supposedly unique one. There are no principled empirical criteria which pick out a preferred set of inertial trajectories as the world lines of the points of absolute space.

This fact is often presented as a problem for the historical substantivalist who is meant to be faced with the following dilemma: either all motion is relative or it is motion with respect to absolute space. The equivalence of inertial frames militates against the existence of absolute space; therefore all motion must be relative. On the other hand there exist empirical criteria which allow one to determine the absolute acceleration of a system—the inertial frames are empirically identifiable—, so motion cannot be analysed simply as relative motion.[5] The story goes that it is only once we learned to do differential geometry and to

[4]We thus side with Rynasiewicz (1996) over DiSalle (1994) on the issue of Newton's substantivalism.

[5]Global inertial frames are operationally identifiable from relative motions alone in *finite* Newtonian universes. If the universe is infinite, one may adopt the perspective of Newton-Cartan theory in which a unique set of *local* inertial frames for every point of spacetime is identifiable. If gravity is regarded as a force and the universe is infinite, then the equivalence principle prevents one from identifying the 'true' global inertial frames because the presence of a uniform gravitational field in a locality is not detectable from local relative motions alone.

view spacetime as a four-dimensional differentiable manifold that we saw how one could deny both that all motion is relative and that a particular inertial frame has a privileged status. An affine connection on the spacetime manifold suffices to determine all the inertial trajectories in a democratic way.[6]

But this is surely too simplistic. Ignorant of the concept of an affine connection, one might still believe that the equivalence of inertial frames undermines the notion of absolute space and at the same time believe that the inertial structure in the world—the existence of a privileged family of reference frames—is not dependent on, and is perhaps 'prior to,' all matter. While Earman might be correct in judging that Poincaré's writings indicate he is captive in a "conceptual box" in thinking that "either the motion of a body is judged solely with respect to other bodies or else with respect to absolute space" (1989, 85), other authors writing at the end of the nineteenth century, such as James Thomson, clearly do not make this mistake (cf. DiSalle 1994, 284).

4 Antirelationist Arguments

Notoriously Leibniz failed to formulate an empirically adequate physics that does away with inertial structure and a temporal metric. To do so has generally been judged to be a hopeless task, for two main reasons. First, the distinction between inertial and non-inertial motion introduced by Newton's laws—a distinction that *prima facie* cannot be accounted for in terms of purely relative motion—is empirically well-founded. This is the moral usually (and correctly) drawn from Newton's bucket experiment and his thought experiment involving the two globes attached by a cord.[7]

This consideration certainly do not prove the impossibility of a relational classical physics. As Mach famously conjectured, the well-confirmed *local* inertial structure displayed by the rotating bucket might actually arise through the bucket's interaction with the rest of the universe. But such a response has force only if an alternative constructive relationist explanation of the phenomena is available. Newtonian mechanics in its standard form *does* have an explanation of the correlation between the concavity of the water in the bucket and the sequence of the water and bucket's relative rotation. Only a relational *theory* predicting the same phenomena would constitute a genuine alternative. This is a point that Earman rightly stresses again and again (1989, e.g., 65, 135).

The second reason why the prospects for a relational classical physics might seem doubtful is well-illustrated by the initial value problem. Many suppose that

[6] Nice examples of the argument can be found in Rynasiewicz (1995*a*, 679) and Belot (1999, 39).

[7] In Newton's hands, the bucket experiment is used to demonstrate the inadequacy of Descartes' relational definition of motion. The globes thought experiment takes Newton's definition of absolute motion (as motion with respect to absolute space) for granted and is designed to demonstrate the extent to which absolute motion can be empirically determined despite the invisibility of absolute space. For two excellent, historically sensitive commentaries on the arguments of Newton's Scholium, see Barbour (1989, Chapter 11) and Rynasiewicz (1995*b*).

if one accepts Leibniz's ontology, one should expect that the relative configuration of an isolated system at an instant and the relative rate of its change should be sufficient to determine its future evolution. This point of view lies behind Poincaré's discussion of absolute space (Poincaré 1952, Chapter 7) and, following Barbour, we will refer to the condition that a truly relational theory should have such initial data as *Poincaré's Criterion*.[8] However, assuming that Newtonian mechanics correctly predicts which future sequences of relative configurations are possible, such information turns out to be insufficient. While the overall position and orientation of the system with respect to an inertial frame is dynamically insignificant, and while its overall velocity within that inertial frame is insignificant also, the rate of change of its orientation—its angular velocity—does have an effect on the relative distances between the system's constituent particles at future times.[9] If the equations of motion of a Newtonian system are re-expressed in terms of relative distances rather than inertial frame positions, the equations can involve 'accidental constants' (encoding the angular momentum of the system) and some will be of greater than second order.

5 Rehabilitating Relationism

There is, however, a subset of the solutions to any Newtonian theory that displays an interesting property. If the total angular momentum of the system as measured in its centre-of-mass inertial frame (the centre-of-mass system or CMS) is zero, then specification of the relative quantities *is* sufficient to determine its future evolution. In this case, the equations of motion in terms of relative distances are all of second order. Relational conclusions were first drawn from this result by Zanstra (1924) who attributes to Föppl the hypothesis that the universe has zero angular momentum. Earman, however, is sceptical that this situation has any relational implications. In his opinion, "it is no big surprise to find that the relationist has an easy time of it when troublesome rotation is absent" (1989, 88).

Fortunately for the relationist, much more can be said. In two recent articles, Gordon Belot discusses the same result in terms of a Hamiltonian formulation of classical mechanics (Belot 1999, Belot 2000). He stresses that the move from the full Newtonian theory to the zero CMS angular momentum solutions "can be given a number of elegant, autonomous formulations—each a fitting competitor to the standard formulation, rather than a parasite" (2000, 571). It will be convenient for what follows to mention some of the details of Belot's discussion. As an illustrative example, we concentrate on the particular system of N gravitating point particles of fixed masses m_i, $i = 1$ to N.

[8]Note that Poincaré does not actually require that one should only need to know the *relative* changes of the relative distances and thus does not confront the *prima facie* non-relational character of Newton's absolute time.

[9]Poincaré, after posing the problem in this way, disputes the conclusion that standard Newtonian dynamics is not relational. His argument is dismissed by Earman (1989, 87) but since it is not obvious that relational initial data can include only relative distances and their *first* derivatives, Poincaré's conclusion deserves more sympathetic attention.

In Hamiltonian mechanics the history of a system is represented by a curve in the system's *phase space* $\mathcal{T}^*\mathcal{Q}$, each point of which encodes the positions and momenta of the particles of the system *in some inertial frame.* The Leibnizian relationist, who believes that only the instantaneous *relative* distances are physically real, might seek to formulate a Hamiltonian theory on the *relative phase space* $\mathcal{T}^*\mathcal{Q}_0$. Here points in the standard $3N$-dimensional *configuration space* $\mathcal{Q}$ which correspond to the same relative configuration, and which are thus transformable into one another by the action of the 6-dimensional Euclidean group E_3, have been identified to form the $(3N-6)$-dimensional *relative configuration space* $\mathcal{Q}_0$. Each point of $\mathcal{T}^*\mathcal{Q}_0$ only encodes information about the relative distances and the relative velocities of the particles.[10] A curve in $\mathcal{T}^*\mathcal{Q}_0$ represents a sequence of such relative configurations.

The conclusion of the discussion so far is that *only* in the case of vanishing CMS angular momentum is the information given by a point in $\mathcal{T}^*\mathcal{Q}_0$ sufficient to predict the evolution of a Newtonian system. Dynamically possible sequences of relative configurations for Newtonian systems with non-vanishing angular momentum cannot be derived from a Hamiltonian theory on $\mathcal{T}^*\mathcal{Q}_0$. But sequences corresponding to systems with zero angular momentum *can* be obtained as the solutions of a "Hamiltonian theory on the relative phase space, employing the canonical symplectic structure on $\mathcal{T}^*\mathcal{Q}_0$, and the projection of the standard Hamiltonian" (Belot 1999, 43). Belot claims, *contra* Earman, that such sequences admit of a "strict relationist interpretation" (1999, 43).[11]

All of this would be of less than academic interest were it not for evidence that our universe is not rotating. In the inertial frame obtained by astronomers from the relative motions of our solar system, the average observed rotation of galaxies with respect to any axis through the sun is less than about 1 arc-sec/century (Schiff 1964). Analysis of the microwave background radiation also suggests that the angular momentum of the universe vanishes (Barrow, Juszkiewicz & Sonoda 1985). Belot regards the non-rotation of the universe as a "contingent fact" which allows a strict relationist interpretation of cosmological applications of Newtonian gravitation to get off the ground. He is thus slightly worried that the substantivalist can accuse the relationist of employing an *ad hoc* manoeuvre in restricting his attention to a proper subset of the full Newtonian theory.

In contrast, we believe that the fact that relational classical mechanics is only consistent with a non-rotating universe is a potential strength of the theory.

[10] Recall footnote 8. These "relative velocities" are still the rates of change of the relative distances *with respect to absolute time.* We return to this point in Section 6.

[11] In an appendix Belot discusses a standard procedure which allows one to factor out symmetries of a given Hamiltonian theory to derive a Hamiltonian theory on a *reduced* phase space. One performs a different reduction for each conserved quantity associated with the symmetries of the original theory. In the case of conserved *linear* momentum, the reduction leads to the same theory in each case, which one can view as a theory defined on $\mathcal{T}^*\mathcal{Q}_{\text{CMS}}$. In the case of angular momentum one obtains a different theory for each value of angular momentum. Moreover, only in the case of zero CMS angular-momentum is the resulting theory a theory on the relative phase space $\mathcal{T}^*\mathcal{Q}_0$. In the other cases the theory is defined on a *sphere bundle* over $\mathcal{T}^*\mathcal{Q}_0$, corresponding to the need to specify the *direction* of angular momentum (given by a point on the 2-sphere) as well as its magnitude. (Cf., also, Belot 2000, 572–3; 581–2.)

From the point of view of the relational theory, the non-rotation of the universe with respect to the operationally definable centre-of-mass inertial frame is *not* contingent; it is something that the theory *predicts*. If it turns out that the measured angular momentum of the universe is not zero then relationism, or at least the variety we have been considering, must be rejected. But if the angular momentum of the universe is found to be zero then, since this is a prediction of the relational theory, it is also something which the relational theory *explains*. From the substantivalist perspective, on the other hand, that the angular momentum of the universe is zero *is* contingent and, in fact, rather extraordinary (given the range of possible values it might have had according to the substantivalist).[12]

In fact, although Belot regards the non-rotation of the universe as a contingent fact, he also admits that "the relationist ploy threatens to convert this into a physical necessity." His considered conclusion is that ultimately the relationist should regard rotating universes as physically possible; they are simply ones "in which space had to be absolute after all". Nonetheless, the relationist will regard such possibilities as less like the actual world than the substantivalist holds them to be: "There is a sense in which worlds governed by the same laws but instantiating distinct ontologies are more remote possibilities than worlds which agree with ours both about laws and about ontology" (1999, 44).

We, on the other hand, are happy to embrace the restriction on physical possibility that relationism appears to involve and we regard such restriction as the prime advantage that relationism has over substantivalism in the pre-relativistic context. We also must confess that it is opaque to us how two possible worlds can differ so fundamentally in their ontologies and yet be held to obey the same laws. Surely it is more natural to rule that such worlds differ with respect to both laws and ontology. Newtonian gravitation and the *relationist interpretation of* the zero-angular momentum fragment of it should be regarded as distinct theories which properly receive distinct mathematical formulations: for example, one might be formulated as a Hamiltonian theory defined on $\mathcal{T}^*\mathcal{Q}$ (or perhaps as theory set in 'Galilean' or 'neo-Newtonian' spacetime), while the other might be formulated as a theory defined on $\mathcal{T}^*\mathcal{Q}_0$.

The relationist must also establish his right to the full theory when it comes to modelling *subsystems* of the universe for these, manifestly, can have non-zero angular momentum. Belot believes that "it is still cogent for them [relationists] to insist that when it comes time to interpret the theory, we need only make sense of its application to the entire universe" (1999, 44). We would add that the cogency of this position is highlighted when one notes that it *follows* from the relational theory (held to describe the universe) that effectively isolated subsystems are correctly described by the full Newtonian theory, subsystems which can thus

[12]This claim can be questioned. Belot reports Frank Arntzenius as suggesting that Newtonian theory also predicts that there will be no observable universal rotation: "as $N \to \infty$, the measure of the set of random distributions of N particles with discernible angular momentum goes to zero (angular momentum requires correlations between velocities)" (Belot 2000, fn. 18). We see a problem with this line of argument. According to Newtonian mechanics, the *actual* initial conditions of the universe, with its low entropy and ubiquitous velocity correlations, must come from a rather special set of measure zero. We do not, after all, live in a heat bath.

have non-zero angular momentum.

Despite all that has been said so far, a certain caution concerning the viability of a relational interpretation of the $\mathcal{T}^*\mathcal{Q}_0$ theory can remain. The source of the scepticism is that this theory has been defined *via* the standard theory. In his (1999), for example, Belot begins with the full Newtonian theory and then factors out its symmetries; it turns out that the result obtained for systems with zero angular momentum can be reinterpreted relationally. There are two responses to this worry. First, one can insist that the historical details of a theory's discovery do not dictate how it should be interpreted.[13] It might be required of a revisionary interpretation that it can accommodate, and perhaps explain, those historical details. But this the relationist interpretation can readily do. As noted above, it accounts for the applicability of the 'substantivalist' theory to subsystems of the universe and this suffices to explain why the substantivalist formulation was discovered first. Second, and more importantly, Barbour and Bertotti's derivation of the relational theory goes via relationally motivated first principles. They do *not* begin with the standard theory or restrict themselves to the non-rotating subset of its solutions. In fact, from their perspective, non-rotation can even be seen as a *novel* prediction.

There is one final aspect of Belot's discussion that we wish to address and it is one that will serve to introduce our discussion of Barbour's work. Belot seeks to distinguish two strains of relationism about motion. According to one, the relationist should seek to implement the programme that originates in Mach's remarks: one should aim to derive the local inertial frames from the relative positions of all the masses in the entire universe and their relative motions (cf. page 5 above). According to the other, one need only construct a theory meeting Poincaré's criterion (in its unmodified form). Belot himself views the first of these as "daunting and poorly defined" (Belot 2000, 570). But can such a distinction between two varieties of relationism be drawn so clearly? In particular, given that the inertial frames are an empirical fact, any empirically adequate theory meeting Poincaré's criterion can either be interpreted as providing a genuinely relational account of them or it will be revealed as little more than a "cheap instrumental rip-off."[14]

In fact, after disavowing the Machian programme, Belot does not remain silent on inertia. In his earlier paper, he comments in a footnote that he suspects that the derivability of a subset of Newtonian solutions from a theory on the relative phase space tells against the anti-relationist argument that is based on the empirical fact of the local definability of non-inertial motion (the argument is

[13] In fact, we believe that Belot would endorse this line of thought. In his most recent paper, we take him to be considerably more sanguine about the viability of the relationist interpretation of the theory he discusses (cf. Belot 2000, fn. 20). He also acknowledges that factoring out the symmetries of the standard $T^*\mathcal{Q}$ is but one way to arrive at the $T^*\mathcal{Q}_0$ theory.

[14] To borrow Earman's now much-quoted phrase. For example, this might well be the correct assessment of Zanstra's 'derivation' of the zero angular momentum fragment of Newtonian theory. Of course, that the fragment satisfies Poincaré's criterion is extremely suggestive in itself. But an autonomous formulation of a theory yielding this fragment as its complete set of solutions seems required before a relationist interpretation of them can get off the ground.

rehearsed above, page 5). His reason is that "the geometrical structures of $\mathcal{T}^*\mathcal{Q}$ and $\mathcal{T}^*\mathcal{Q}_0$ encode an awful lot of information about 'inertial structure' without appealing to any notions of absolute motion" (1999, fn. 5). Belot is of course right that the $\mathcal{T}^*\mathcal{Q}_0$ theory undermines the anti-relationist argument. But we believe that inertial structure does not enjoy the same status in both the $\mathcal{T}^*\mathcal{Q}$ and $\mathcal{T}^*\mathcal{Q}_0$ theories. In particular, while the $\mathcal{T}^*\mathcal{Q}$ *does* employ, if tacitly, a notion of absolute motion, the $\mathcal{T}^*\mathcal{Q}_0$ theory, *when understood correctly*, does not.

In his (2000) Belot returns to the subject, again in a footnote which we quote in full:

> What is the connection between this brand of relationism, and the sort more often considered by philosophers? How *does* this theory account for inertial forces in terms of relational variables? As in any simple mechanical system, here the inertial effects derive from the metric on configuration space, which is inherited from the metric on standard configuration space, which is induced by the Euclidean metric on physical space. We can talk about the structure of the reduced configuration space, and its relation to the structure of the physical space in which the particles move, without referring to an affine structure on spacetime. This is one of the reasons why questions about classical mechanics are sometimes better posed in terms of the structure of Hamiltonian systems, rather than in terms of 'spacetime theories'. (2000, 573, fn. 29)

But is reference to the affine structure of spacetime so easily avoided? In particular, how should one understand "*the structure of the physical space in which the particles move*"? The particles *move* in the *persisting* relative space of an inertial frame (or even, to adopt a more literal interpretation of the formalism, in persisting absolute space). The structure of *this* space is exactly (in part) affine structure on spacetime.[15] If the only way to understand the metric of the relative configuration space is as "inherited" from the metric on the standard configuration space then the $\mathcal{T}^*\mathcal{Q}_0$ Hamiltonian theory is starting to look decidedly less relational than it at first appeared.

Fortunately, there is an alternative. When introducing the idea of a Hamiltonian theory defined on the relative phase space, Belot notes that there are two ways of proceeding. One can *first* equip the relative configuration space with a metric is such a way that the solutions of the resulting Hamiltonian theory coincide with zero angular momentum solutions of a Newtonian theory. Alternatively

[15] The same point can be made in the context of the Lagrangian rather than the Hamiltonian formulation of classical dynamics, formulated in $\mathcal{QT}$, the space formed by adjoining the space of absolute times T to the standard configuration space $\mathcal{Q}$. Pairs of curves in $\mathcal{QT}$ can be assigned different values of the action despite corresponding to the same sequence of relative configurations. This is how the equilocality relation of Newtonian spacetime is stipulated to make a dynamical difference. We therefore hold that it is quite misleading to claim that no appeal to notions of absolute motion is made in the $\mathcal{T}^*\mathcal{Q}$ theory. (Recall that Lagrangian mechanics associates with every curve between any two appropriate points (q_1, t_1) and (q_2, t_2) in $\mathcal{QT}$ a quantity called the action $S = \int_{t_1}^{t_2} L(q, \dot{q}, t)dt$. The curve which represents the genuine dynamical history connecting the two points is one for which $\delta S = 0$.)

one can *start* with the Newtonian theory, restrict ones attention to the subspace of the phase space for which the angular momentum is zero and *then* factor out the symmetries of the original theory.[16]

It is the first of these perspectives which the relationist should embrace. Doing so would appear to be a prerequisite to viewing the $\mathcal{T}^*\mathcal{Q}_0$ theory as providing a relational *reduction* of the inertial frames rather than as encoding the remnants of non-relational structure. This is a first step. One also needs to give a positive relational account of the metric of the relative configuration space and the equilocality relation it encodes.[17] To see how this can be achieved one needs to consider Barbour's work. We present the central ideas in the remainder of this paper. We deal first with issues concerning the temporal metric and return to the issue of equilocality in Section 7.

6 Dynamics on the relative configuration space

There is one important respect in which the Hamiltonian theory on $\mathcal{T}^*\mathcal{Q}_0$ discussed by Belot fails to be relational. Points in the $(3N-6)$-dimensional cotangent spaces $\mathcal{T}_q^*$ associated with each point of $q \in \mathcal{Q}_0$ encode the rates of change of the relative distances *with respect to absolute time.* Only $3N-7$ numbers are required to specify the *relative* rates of change of the relative distances. That is to say, the natural 'Leibnizian' initial data consist solely of a point and a *direction* in $\mathcal{Q}_0$. Possible Leibnizian histories consist of curves in $\mathcal{Q}_0$. Just as the Leibnizian relationist believes that, from a kinematical point of view, there is no privileged embedding of the curve in the larger space $\mathcal{Q}$, so too he believes that, from the point of view of fundamental kinematics, there is no preferred parametrization of the curve.[18]

In constructing a genuinely relational theory, the Lagrangian framework for mechanics is arguably more apt than the Hamiltonian. Whereas standard Newtonian theory can be formulated as an action principle on $\mathcal{Q}\mathcal{T}$, the relationist seeks a theory that can be formulated as a variational principle on $\mathcal{Q}_0$ alone. The initial data of such a theory will meet Poincaré's Criterion modified to take account of the fact that the relationist should not postulate a temporal metric.

One family of theories of this type has been repeatedly rediscovered through-

[16]One also needs to fix on an arbitrary value of linear momentum to define the appropriate subspace of the original phase space. For more details see Belot (2000, 572–3) and references therein.

[17]Perhaps it is open to the relationist to take the $\mathcal{Q}_0$ metric as primitive. The point that he should not understand it as encoding structure of the larger space, $\mathcal{Q}$, remains valid.

[18]We use phrases such as "Leibnizian relationist" to denote someone who believes that the only objective spatial and temporal quantities are instantaneous (Euclidean) relative distances. This is the classical relationist familiar from the contemporary philosophical literature. (Recall that for present purposes we simply assume that such a relationist is correct in seeking a theory meeting Poincaré's criterion.) What contact such a position (which, for example, treats spatial and temporal relations very differently) has with Leibniz's philosophical principles is a moot question (see Saunders forthcoming).

out the century.[19] As the rediscovery indicates, the theories never became widely known; but their treatment at the hands of Barbour and Bertotti in the 1970s is discussed by Earman (1989, Chapter 5). The theories involve a Lagrangian defined directly in terms of the particles' masses and relative distances. The particular action discussed in Barbour & Bertotti (1977) is:

$$S_{\mathrm{BB1}} = \int \mathrm{d}\lambda \sqrt{-VT_{\mathrm{BB1}}},$$
$$\text{where} \quad V = -\sum_{i<j} \frac{m_i m_j}{r_{ij}} \quad \text{and} \quad T_{\mathrm{BB1}} = \sum_{i<j} \frac{m_i m_j}{r_{ij}} \left(\frac{\mathrm{d}r_{ij}}{\mathrm{d}\lambda}\right)^2 . \tag{6.1}$$

V is proportional to the standard Newtonian gravitational potential which is already defined solely in terms of relative distances. T_{BB1} is a relational or 'Machian' kinetic energy term. λ is an *arbitrary*, monotonically increasing parameter labelling the points of trial curves in $\mathcal{Q}_0$. The action is invariant under the reparametrization $\lambda \to \lambda' = f(\lambda)$, $\frac{\mathrm{d}f}{\mathrm{d}\lambda} > 0$.

Newtonian-like behaviour can be derived from this action principle and deviations from standard Newtonian mechanics can be tailored to achieve some welcome results. Schrödinger, for example, showed how the theory can yield an anomalous advance of the perihelia of the planets exactly matching that predicted by general relativity. Ultimately, though, the theory is empirically inadequate, most strikingly in its prediction of mass-anisotropy effects. The inertial mass of a body will be larger in the direction towards the centre of mass of the Galaxy than in perpendicular directions.

Additionally a serious problem faces *any* theory defined directly in terms the relative distances between point objects: how can such a theory be extended to the field ontology of modern physics? There are obvious field analogues for the r_{ij}s (see Pooley 2001, Section 2.1). What is missing is an *a priori* way of identifying some part of a matter field at one instant with a part of the matter field at a later instant: there are no field analogues of $dr_{ij}/d\lambda$. In the next section we describe a different approach, discovered by Barbour and Bertotti (1982), to constructing a relational particle theory. Their purpose, in part, was to devise a framework that could be generalized to field theories.

If T_{BB1} is replaced in S_{BB1} by the standard kinetic energy term,

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda} \cdot \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda}, \tag{6.2}$$

one obtains an action principle defined on $\mathcal{Q}$ rather than $\mathcal{Q}_0$; $\mathbf{x}_i$ is the i^{th} particle's position in an inertial frame. This action principle is actually a special case of Jacobi's Principle which has as solutions the standard Newtonian orbits of a

[19] Noteworthy examples are Reissner (1914), Schrödinger (1925), Barbour (1974*a*), Barbour & Bertotti (1977) and Assis (1989). For further discussion see Barbour & Pfister (1995) and Barbour (1999). These theories, with the exception of Barbour and Bertotti's, do not question Newton's absolute time and are thus effectively formulated in $\mathcal{Q}_0 T$.

system of gravitating particles of one particular energy, in this case zero energy. The full range of Jacobi's Principle actions representing a system of Newtonian gravitating particles is given by:

$$S_{\text{Jac}} = \int \mathrm{d}\lambda \sqrt{F_E T}, \qquad \text{where} \quad F_E = E - V_G = E + G \sum_{i<j} \frac{m_i m_j}{r_{ij}} \tag{6.3}$$

and where E is the (constant) total energy of the system.

The conventional interpretation of a variational principle based on such an action is that it is a tool which can be used to find the orbits in $\mathcal{Q}$ of the truly fundamental variational principle $\delta S = 0$ for $S := \int \mathrm{d}t(T - V)$, defined on $\mathcal{QT}$. For the latter, where duration enters at the level of kinematics, the system is not restricted to any particular energy. However, an alternative interpretation is possible. An action principle based on (6.3) can be interpreted as fundamental when applied to the entire universe. It will, of course, involve a particular value of E which is now to be regarded as a constant of nature. Newton's absolute time is then seen as having a *dynamical* origin.

The equations of motion which follow from the variational principle $\delta S_{\text{Jac}} = 0$ are:

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\sqrt{\frac{F_E}{T}} m_i \frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda}\right) = \sqrt{\frac{T}{F_E}} \frac{\partial F_E}{\partial \mathbf{x}_i}. \tag{6.4}$$

These reduce to a simple and familiar form for a particular choice of λ. If it is chosen so that $T = F_E$ then, writing t for this special choice of λ, (6.4) becomes:

$$m_i \frac{\mathrm{d}^2\mathbf{x}_i}{\mathrm{d}t^2} = -\frac{\partial V}{\partial \mathbf{x}_i}, \tag{6.5}$$

i.e., Newton's second law. One should not see the imposition of $T = F_E = E - V_G$ as an application of the energy conservation law, derivable from a variational principle in $\mathcal{QT}$. From the current perspective it *defines* a temporal metric.

This special choice of parameter $\lambda = t$ does not merely simplify the equations. It follows from the structure of the variational principle, here taken to describe the universe, that effectively isolated subsystems will also obey a Jacobi principle. However, unlike the universe as a whole, where E is regarded as a *fixed* fundamental constant, these subsystems can in principle have any constant value of E. If one considers the temporal parameters defined by imposing $T = F_E$ for each such subsystem, one finds that they differ by at most a constant of proportionality. It is only if a time parameter is chosen in this way that the times defined by different subsystems of the universe are consistent. In Barbour's phrase, $\lambda = t$ is the "uniquely useful" choice of time parameter. This point is closely related to another which is most significant when it comes to an evaluation of the viability of the relationist interpretation: if (6.3) did represent the basic law governing the universe, then it is clear why one might erroneously come to believe in the existence of absolute time from the study of effectively isolated subsystems.

Jacobi's principle is actually a *geodesic* principle. The kinetic term (6.2) defines a flat Riemannian metric on $\mathcal{Q}$. $\sqrt{F_E}$ is a conformal factor which transforms the flat metric into a new, non-trivial metric. The extremal curves of $\delta S_{\rm Jac} = 0$ are the geodesics of this metric.

7 Intrinsic Particle Dynamics

We now return to the problem of constructing a theory on $\mathcal{Q}_0$ in a way which can be generalized to field theories. Consider a relative configuration of N point particles. $\frac{1}{2}N(N-1)$ Euclidean relative distances r_{ij} define the configuration. (Only $3N-6$ of these are independent since they must satisfy the algebraic relationships of Euclidean geometry.) These relative distances can be *represented* in terms of a Cartesian coordinate system: if $\mathbf{x}_i$ is the position vector of the i^{th} particle in this coordinate system, then one requires that $|\mathbf{x}_i - \mathbf{x}_j| = r_{ij}$. The 'positioning' of the relative configuration with respect to the origin and coordinate axes of the coordinate system is entirely *arbitrary*: we emphasize that the $\mathbf{x}_i$ are *not* particle positions in some inertial frame. All coordinate axes that are transformable into the original ones by a member of the improper Euclidean group encode the same relative distances.[20]

Now consider two such relative configurations which differ intrinsically. These correspond to two points q_0^1 and $q_0^2 \in \mathcal{Q}_0$, $q_0^1 \neq q_0^2$. In order to set up a geodesic principle on $\mathcal{Q}_0$ one requires a measure of the *intrinsic* difference between q_0^1 and q_0^2. Let us coordinatize q_0^1 and q_0^2 with two Cartesian coordinate grids, as described above, and consider the function

$$D = \sqrt{\frac{1}{2}\sum_{i=1}^{N} m_i \delta\mathbf{x}_i . \delta\mathbf{x}_i}, \tag{7.1}$$

as a candidate measure of the difference between the two configurations. Here $\delta\mathbf{x}_i$ is defined as follows. If $\mathbf{x}_i$ is the position of the i^{th} particle with respect to the coordinate system freely chosen to represent the relative distances of q_0^1 and $\mathbf{x}_i'$ is the position of the i^{th} particle with respect to the different coordinate system freely chosen to represent the relative distances of q_0^2, then $\delta\mathbf{x}_i := \mathbf{x}_i' - \mathbf{x}_i$.[21]

D clearly does not give a measure of the intrinsic difference between the two relative configurations, for its value depends on the arbitrary placement of the coordinate grids used to describe the configurations. However, D can be used to define an *objective, coordinate-independent difference.* There is always complete freedom in the positioning of the first coordinate grid relative to the first configuration. However, given such a choice of coordinates for q_0^1, the second coordinate grid can be rigidly shifted relative to the second configuration q_0^2 so that (7.1) is minimized. If coordinates for q_0^2 are initially chosen so that the $\delta\mathbf{x}_i$

[20] The improper Euclidean group is obtained from the Euclidean group by the addition of the discrete transformation of parity inversion: $\mathbf{x}_i \to -\mathbf{x}_i$.

[21] Note that at this stage, there is simply no sense in which one could refer them to the same coordinate system. q_0^1 and q_0^2 are autonomous entities which, we are assuming, do not live in some substantival embedding space or spacetime.

are small, this shifting can be represented as:

$$\begin{aligned}\mathbf{x}_i' \to \left(1 - \textstyle\sum_{\alpha=1}^{6}\varepsilon_\alpha O_\alpha\right)\mathbf{x}_i' &= \left(1 - \textstyle\sum_\alpha \varepsilon_\alpha O_\alpha\right)(\mathbf{x}_i + \delta\mathbf{x}_i) \\ &\approx \mathbf{x}_i + \delta\mathbf{x}_i - \textstyle\sum_\alpha \varepsilon_\alpha O_\alpha \mathbf{x}_i\end{aligned} \tag{7.2}$$

$$\Rightarrow \delta\mathbf{x}_i \to \delta\mathbf{x}_i - \textstyle\sum_\alpha \varepsilon_\alpha O_\alpha \mathbf{x}_i, \tag{7.3}$$

where the O_αs are the six generators of the Euclidean group. The minimization of (7.1) by this procedure can then be written:

$$\delta D = 0, \quad D = \sqrt{\frac{1}{2}\textstyle\sum_{i=1}^{N} m_i \left(\delta\mathbf{x}_i - \sum_\alpha \varepsilon_\alpha O_\alpha \mathbf{x}_i\right)\cdot\left(\delta\mathbf{x}_i - \sum_\alpha \varepsilon_\alpha O_\alpha \mathbf{x}_i\right)}, \tag{7.4}$$

where the minimization is with respect the auxiliary variables ε_α.

The process defines a metric on $\mathcal{Q}_0$. The 'distance' between q_0^1 and q_0^2 is given by

$$D_{\text{Intr}} = \sqrt{\frac{1}{2}\textstyle\sum_i m_i \left(\delta\mathbf{x}_i - \sum_\alpha \varepsilon_{0\alpha} O_\alpha \mathbf{x}_i\right)\cdot\left(\delta\mathbf{x}_i - \sum_\alpha \varepsilon_{0\alpha} O_\alpha \mathbf{x}_i\right)}, \tag{7.5}$$

where the $\varepsilon_{\alpha 0}$ are the minimizing values of the auxiliary variables. It is already a curved, non-trivial metric,[22] and if it is used in the formulation of a geodesic principle on the relative configuration space, it accounts for the inertial (uniform, rectilinear) motions of N force-free bodies. Note that, in contrast to the standard treatment of Newton's first law, the masses of the particles here play an essential role in defining such inertial behaviour. This is because the inertial motions are automatically constrained by the above *best matching* procedure to have zero total angular momentum relative to the centre-of-mass frame, as will become apparent.

Of course, what is desired is something less trivial than pure inertial motion. Just as the standard kinetic metric on $\mathcal{Q}$ can multiplied by a conformal factor to yield Jacobi's geodesic principle (6.3), multiplying D_{Intr} by a conformal factor defines a new action principle on $\mathcal{Q}_0$. Specifically, consider:

$$\begin{aligned}&\delta S_{\text{BB2}} = 0, \qquad S_{\text{BB2}} = \int \mathrm{d}\lambda \sqrt{F_E T_{\text{BB2}}}, \\ &T_{\text{BB2}} = \frac{1}{2}\sum_{i=1}^{n} m_i \left(\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda} - \textstyle\sum_\alpha a_\alpha(\lambda)\, O_\alpha \mathbf{x}_i\right)\cdot\left(\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda} - \textstyle\sum_\alpha a_\alpha(\lambda)\, O_\alpha \mathbf{x}_i\right),\end{aligned} \tag{7.6}$$

with F_E as given in (6.3).[23] This variational principle is formally a principle on a $3N$-dimensional space $\mathcal{Q}$. However, the points of this $\mathcal{Q}$ do *not* represent the

[22] See Gergely & McKain (2000) for a discussion of some of the geometric properties of the metric D_{Intr} for three bodies.

[23] The reader will have noticed that our notation for the auxiliary variables has changed: in (7.4) they were ε_α, in (7.6) they are $a_\alpha(\lambda)$. The λ-dependent correction term in (7.6) must be such that, when integrated along a curve joining two points of $\mathcal{Q}$, $q_1(\lambda)$ and $q_2(\lambda)$, it yields an element of the Euclidean group. For two nearby points $q_1(\lambda)|_{\lambda=\lambda_0}$ and $q_2(\lambda)|_{\lambda=\lambda_0+\delta\lambda}$, one has $\sum_\alpha \varepsilon_{0\alpha} O_\alpha \mathbf{x}_i \approx \sum_\alpha a_{0\alpha}(\lambda) O_\alpha \mathbf{x}_i \delta\lambda$.

inertial frame positions of the particles. Rather, as explained above, they are simply a convenient way of representing the relative distances. Note also that it is a significant feature of this construction that the masses that occur in F_E are the same as those that occur in T_{BB2}: as in the standard treatment of Newtonian gravity, the proportionality of gravitation charge and inertial mass is postulated rather than explained.

For each trial curve $\mathbf{x}_i(\lambda)$ in $\mathcal{Q}$, S_{BB2} is first minimized with respect to the a_α. This gives six functions $a_{0\alpha}(\lambda)$ for each curve. The value of S_{BB2} associated with every curve is then only based on the intrinsic differences between the constituent configurations; all curves in $\mathcal{Q}$ which correspond to the same sequence of relative configurations are assigned the same value of S_{BB2}. The standard variational procedure then picks out the unique sequence of such relative configurations between two given relative configurations which is to constitute the genuine physical history.

What do the solutions of this theory look like? In Barbour's words,

> the really remarkable and ironic fact is that the relative motions predicted by the BB2…model are identical to the relative motions in Newtonian mechanics for a system having vanishing centre-of-mass angular momentum… and one fixed total energy E. (Barbour 1995, 224)

To see why, recall the definition of D_{Intr} (equation 7.5). This involved the arbitrary coordinatization of two relative configurations which differ intrinsically and then the adjustment of the coordinate system of the second of the configurations to minimize a certain quantity (D of equation 7.1). This "best matching" adjustment *defines* an equilocality relation between the relative spaces of the two configurations: points are to be regarded as equilocal when they are given the same coordinates in pairs of coordinate systems which minimize D.

(7.5) is written in terms of two arbitrary coordinate systems, hence the appearance of the $\sum_\alpha \varepsilon_{0\alpha} O_\alpha \mathbf{x}_i$ correction terms in order to achieve minimization. If the second configuration is recoordinatized in terms of the coordinate system found by the minimization procedure, these terms vanish. If such coordinate systems are chosen for an entire sequence of relative configurations, the expression for the action assigned by (7.6) becomes *formally* identical to that of Jacobi's Principle.

Call the curve in $\mathcal{Q}$ which coordinatizes a particular sequence of relative configurations in this way the "stacked curve". Now suppose that a particular stacked curve in $\mathcal{Q}$ minimizes the standard Jacobi's Principle action (6.3). A little reflection shows that it also minimizes (7.6); for when the second variation of (7.6) takes place we can afford to consider only each stacked curve representative of a possible sequence of relative configurations. The action that (7.6) assigns to these is identical to the action Jacobi's Principle assigns to them. However, when the action of Jacobi's Principle is mimimized *all* relevant curves in $\mathcal{Q}$ are considered, including those considered in the (7.6) variation. So a stacked solution of (6.3) minimizes (7.6).

But not all curves which minimize (6.3) minimize (7.6), for not all curves which minimize (6.3) are stacked curves. In fact, the variation with respect to the $a_\alpha(\lambda)$ implies that, with respect to the stacked coordinate system, the linear and angular momentum of the system vanishes (see Barbour & Bertotti (1982, 298–9) and Barbour (1994, 2862–3)). The solutions to (7.6) thus match those of the Hamiltonian theory discussed by Belot *for the fixed energy E*. But just as with Jacobi's principle, the variational principle based on (7.6) also gives us a dynamical understanding of Newton's absolute time: it is reduced to an emergent feature of the dynamical evolution of purely relational entities. The theory also give us a relational reduction of the inertial frames.

It is worth stressing this last point. At the end of Section 5, we noted the need for a way to understand the metric on $\mathcal{Q}_0$ as genuinely relational. Best matching provides exactly this. Recall the trial measure D of the difference between two relative configurations (equation 7.1). We noted that it depended systematically on the arbitrary way the two configurations had been coordinatized. By minimizing (7.1) to obtain a *coordinate independent* measure of the difference between two relative configurations, the relationist is defining a measure that depends *solely* on the intrinsic nature of the objects being compared. This is stark contrast to the Newtonian solution. Here a primitive preferred coordinatization of the two configurations is simply *postulated*: to obtain the Newtonian measure of the difference between the configurations, one chooses coordinate systems adapted to the *same inertial frame*. This highlights both the way in which a theory based on the standard metric on $\mathcal{Q}$ does involve primitive notions of absolute motion and space*time* structure and the way in which a theory based on best matching does without them.

Finally, it might be worth noting that the form of the action principle given in (7.6) is by no means the only one that meets Poincaré's criterion and that is thus potentially compatible with a relationist treatment of classical gravitation. Adding a term to the square-root integrand in the expression for S_{BB2}, provided that this new term is a total derivative with respect to λ of some function of the $\mathbf{x}_i$ and λ, will make no change to the ensuing equations of motion. This modified action principle will not necessarily correspond to a geodesic principle on $\mathcal{Q}_0$, and nor, in general, will the modified action be reparameterization invariant.

8 Conclusion

The foregoing makes it clear, we hope, that Barbour and Bertotti's intrinsic particle dynamics is a genuinely relational theory with certain advantages over the full, substantival Newtonian theory. It is completely consistent with the empirically identifiable inertial equilocality relation and temporal metric; indeed it reduces these aspects of the world to emergent features of the dynamical evolution of the relative configurations of the entire universe alone. In this way it succeeds in implementing Mach's idea that the local inertial properties of a test body are dependent on the remainder of the masses in the universe. Additionally, the fact that it yields only the zero angular momentum subset of full Newtonian theory means that, potentially, it has more predictive and explanatory power than standard classical dynamics. So far the prediction that the universe is not rotating is consistent with the evidence.

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